

Year 12 Mathematics Specialist 2018

Test Number 2: Functions and Graph Sketching

Resource Free

Name: _____

Teacher: DDA

Marks: 45

Time Allowed: 45 minutes

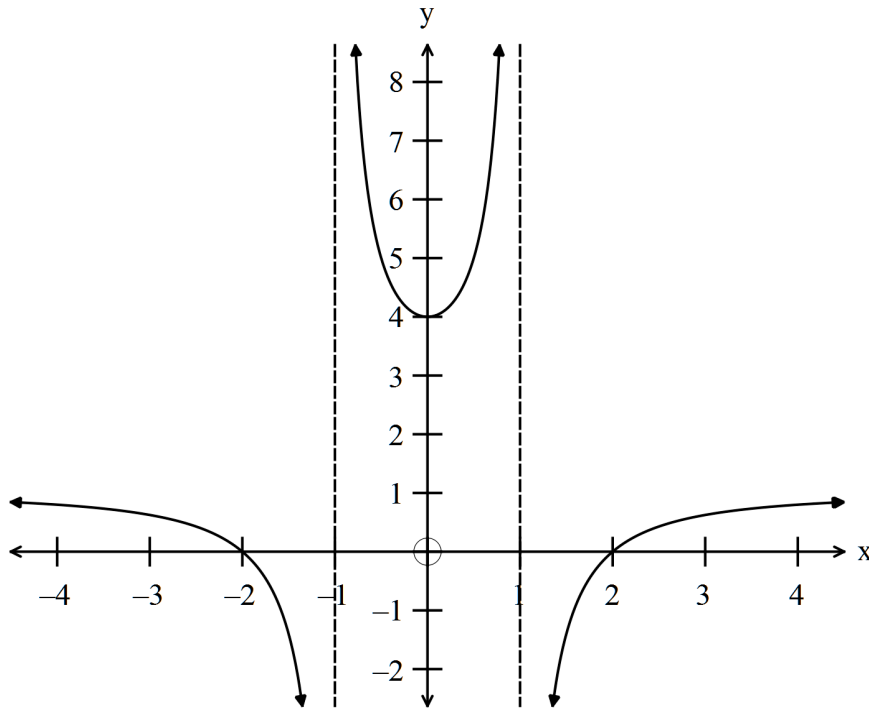
Instructions: You **ARE NOT** permitted any notes or calculator. Show your working where appropriate remembering you must show working for questions worth more than 2 marks.

Question 1

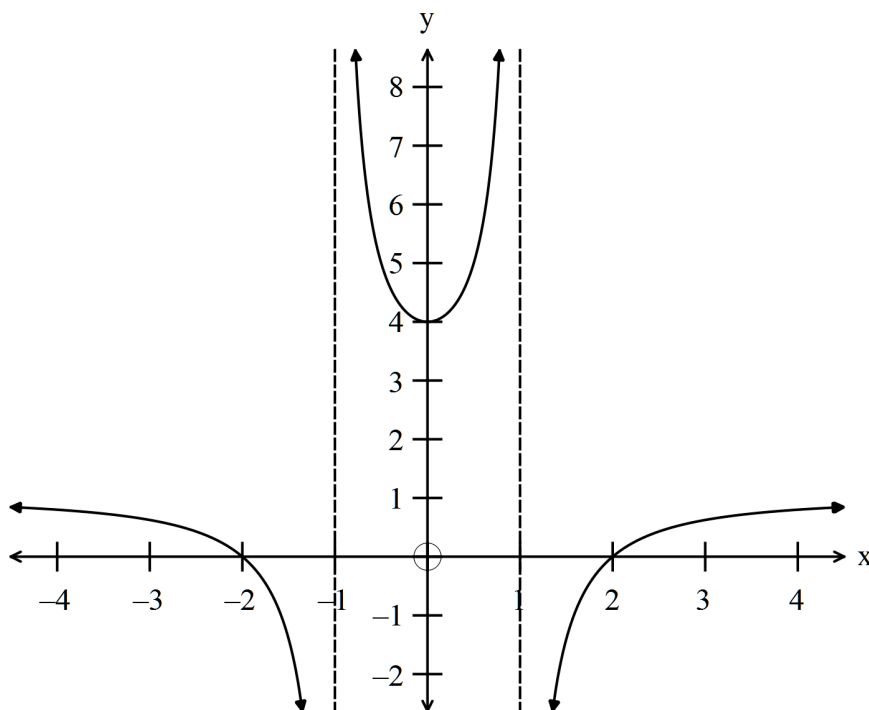
[5 marks]

(a) Given the sketch of the function $f(x) = \frac{(x^2 - 4)}{(x^2 - 1)}$ sketch

(i) $y = |f(x)|$ (2)



(ii) $y = \frac{1}{f(x)}$ (3)



Question 2**[10 marks]**

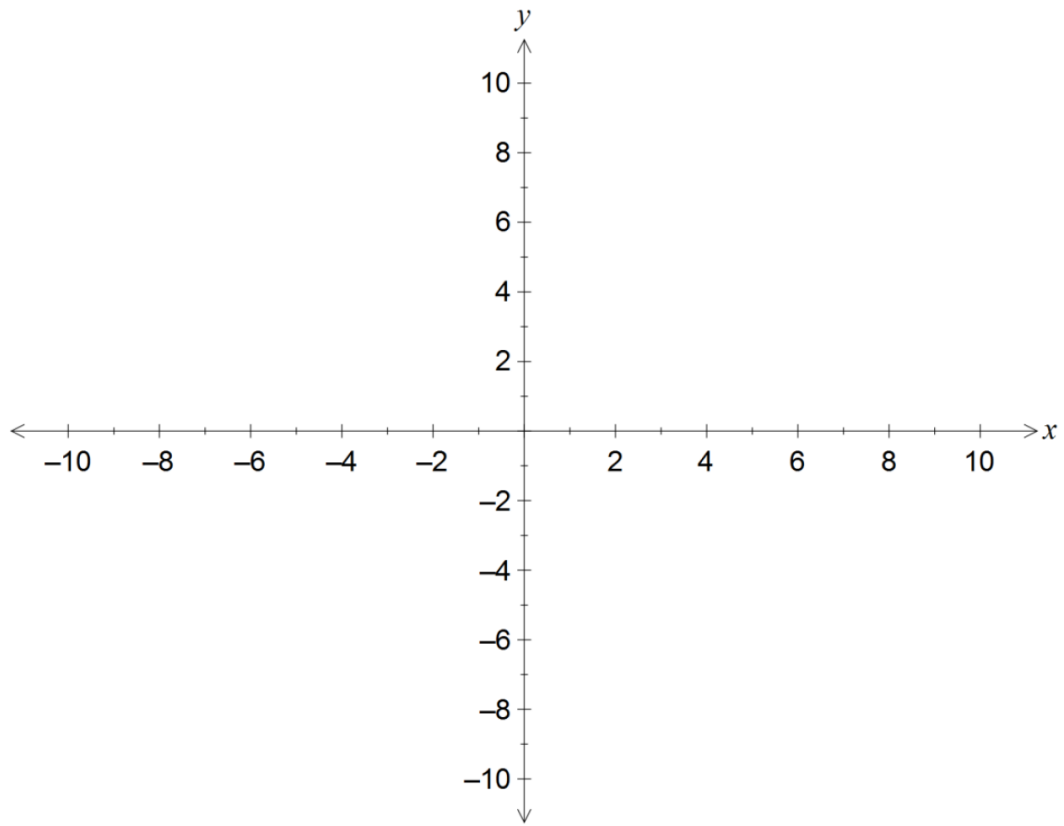
The function f is defined by $f(x) = \frac{x^2 - 6x + 9}{x - 2}$.

The first derivative of f is $f'(x) = \frac{x^2 - 4x + 3}{(x - 2)^2}$.

- (a) State the coordinates of the y -axes intercept. (1 mark)
- (b) Determine the coordinates of the stationary points of the graph of $y = f(x)$. (3 marks)
- (c) Determine the equations of all asymptotes of the graph of $y = f(x)$. (3 marks)

(d) Sketch the graph of $y = f(x)$ on the axes below.

(3 marks)



Question 3**[9 marks]**

Consider the function defined by $f(x) = \frac{1}{2x-1}$.

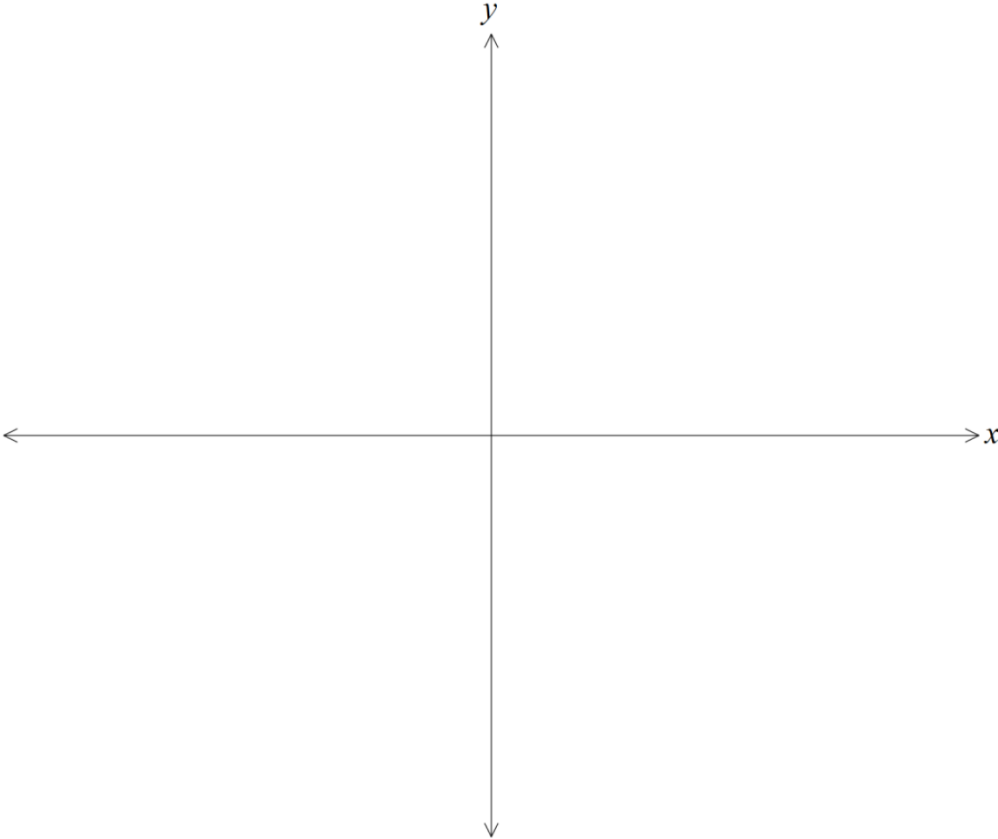
(a) State the natural domain for the function $f(x)$. (1 mark)

(b) Determine the inverse of $f(x)$. (2 marks)

(c) Determine the composite function $f \circ f(x)$, expressing your answer as a single rational function. (3 marks)

(d) Sketch the graph of $y = f \circ f(x)$ on the axes below.

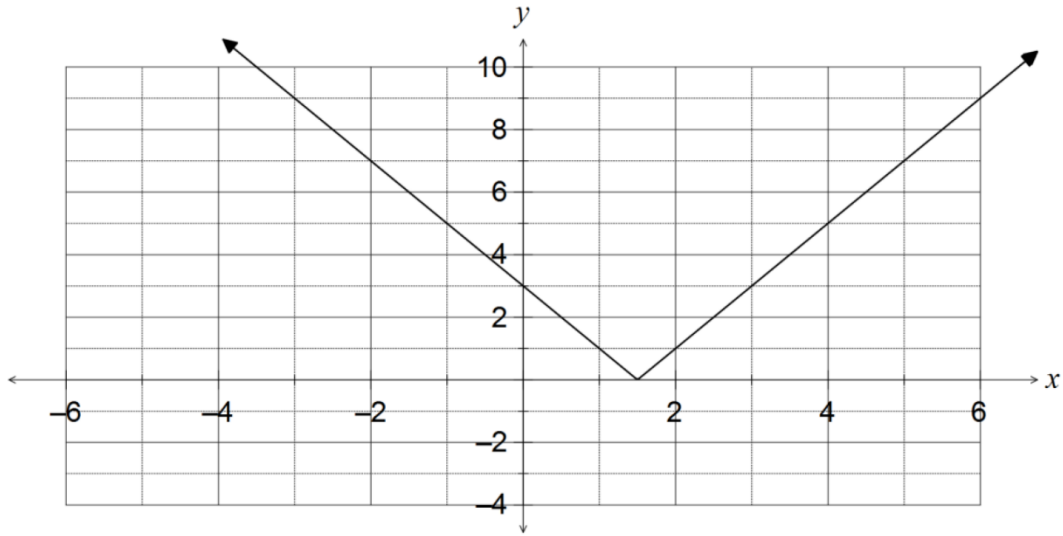
(3 marks)



Question 4

[6 marks]

The graph of $y = |f(x)|$ is shown, where $f(x) = 2x - 3$.



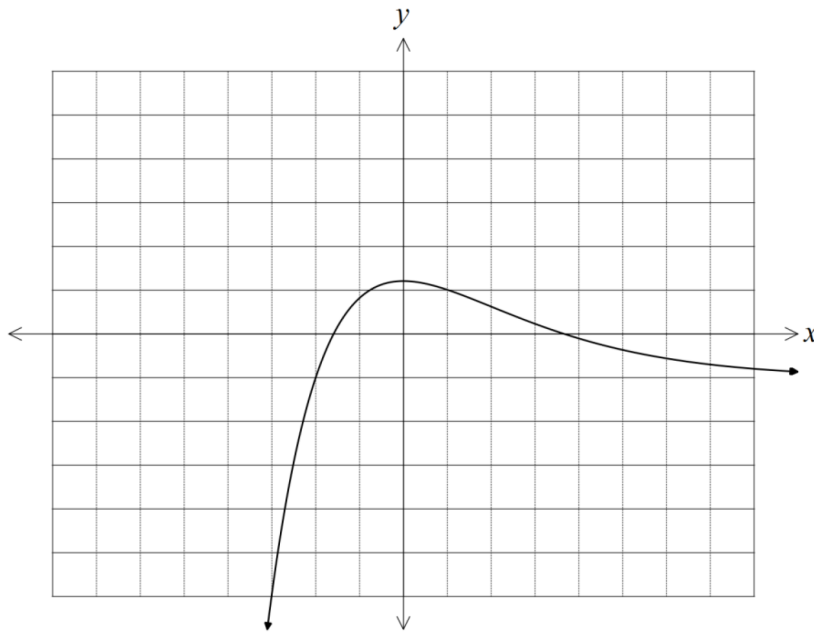
- (a) Add the graph of $y = |g(x)|$ to the axes above, where $g(x) = (x - 2)^2 - 4$. (2 marks)
- (b) Solve $|f(x)| = |g(x)|$. Give exact answers. (4 marks)

Question 5

[4 marks]

- (a) The graph of $y = f(x)$ is shown below.
On the same axes, sketch the graph of $y = f(|x|)$

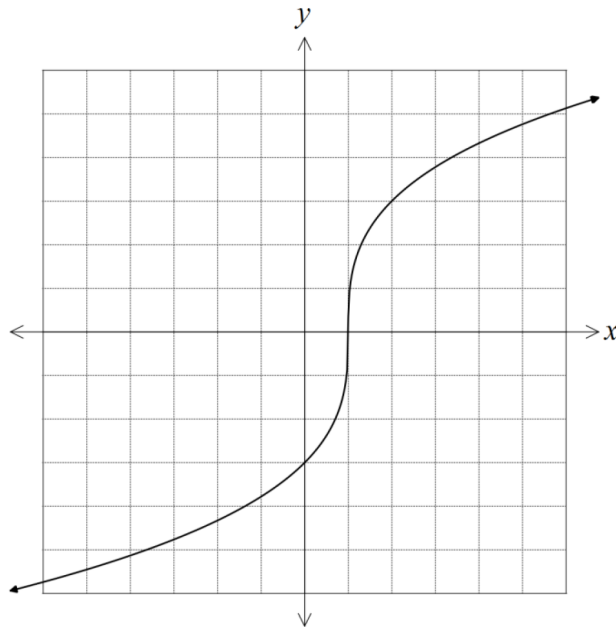
(2 marks)



- (b) The graph of $y = h(x)$ is shown below.

On the same axes, sketch the graph of the inverse of h , $y = h^{-1}(x)$.

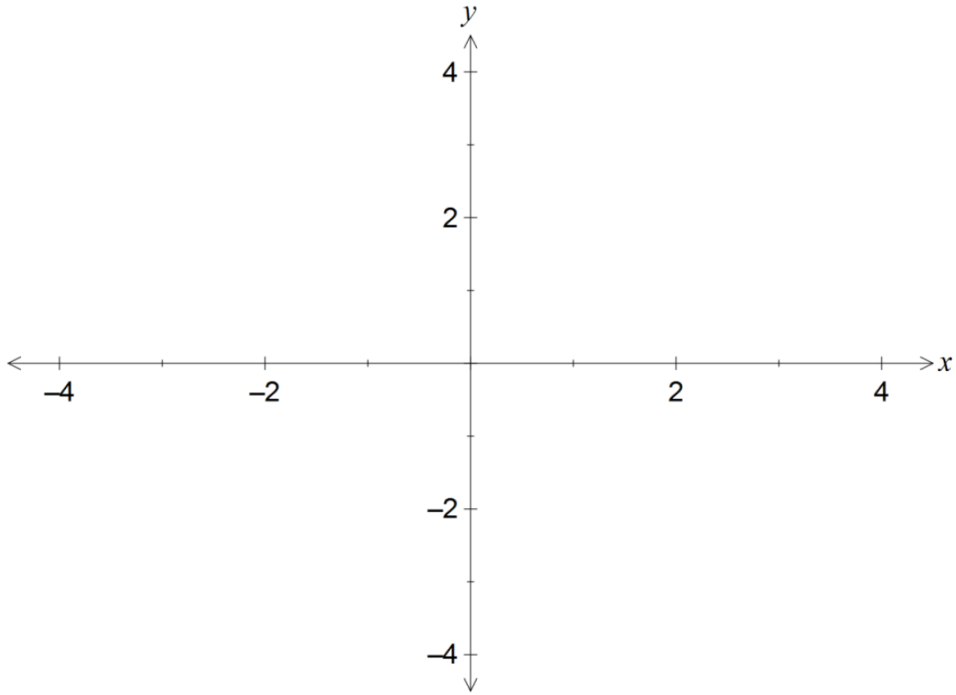
(2 marks)



Question 6

[7 marks]

On the axes below sketch the graphs of $y = 2 - |x + 1|$ and $y = |3x + 2|$, and hence solve the inequality $2 - |x + 1| > |3x + 2|$.



Question 7**[4 marks]**

For each of the following determine, with reasons, whether they are a 1-1 function, a many-to-one function or neither.

$$f(x) = x^3 - x, \quad g(x) = \frac{1}{5} - x, \quad x = y^2$$